Aplikasi Sistem Persamaan Linier dalam Persoalan Dunia Nyata (*real world problem*)

IF2123 Aljabar Geometri

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- Modeling with Linear Systems

Sumber: *College Algebra*, Fifth Edition,
James Stewart Lothar Redlin Saleem Watson

Modeling with Linear Systems

- Linear equations—often containing hundreds or even thousands of variables—occur frequently in the applications of algebra to the sciences and to other fields.
  - For now, let’s consider an example that involves only three variables.

E.g. 1—Nutritional Analysis

- A nutritionist is performing an experiment on student volunteers.
  - He wishes to feed one of his subjects a daily diet that consists of a combination of three commercial diet foods:

    MiniCal
    LiquiFast
    SlimQuick

- For the experiment, it’s important that, every day, the subject consume exactly:
  - 500 mg of potassium
  - 75 g of protein
  - 1150 units of vitamin D

- The amounts of these nutrients in one ounce of each food are given here.

	MiniCal	LiquiFast	SlimQuick
Potassium (mg)	50	75	10
Protein (g)	5	10	3
Vitamin D (units)	90	100	50

  – How many ounces of each food should the subject eat every day to satisfy the nutrient requirements exactly?

- Let x, y, and z represent the number of ounces of MiniCal, LiquiFast, and SlimQuick, respectively, that the subject should eat every day.

- This means that he will get:
  - 50x mg of potassium from MiniCal
  - 75y mg from LiquiFast
  - 10z mg from SlimQuick

- This totals $50x + 75y + 10z$ mg potassium.

	MiniCal	LiquiFast	SlimQuick
Potassium (mg)	50	75	10
Protein (g)	5	10	3
Vitamin D (units)	90	100	50

- Based on the requirements of the three nutrients, we get the system

$$\begin{cases} 50x + 75y + 10z = 500 & \text{Potassium} \\ 5x + 10y + 3z = 75 & \text{Protein} \\ 90x + 100y + 50z = 1150 & \text{Vitamin D} \end{cases}$$

- Dividing the first equation by 5 and the third by 10 gives the system

$$\begin{cases} 10x + 15y + 2z = 100 \\ 5x + 10y + 3z = 75 \\ 9x + 10y + 5z = 115 \end{cases}$$

  - We can solve this using Gaussian elimination.
  - Alternatively, we could use a graphing calculator to find the reduced row-echelon form of the augmented matrix of the system.

Solution: $x = 5$, $y = 2$, $z = 10$

- Every day, the subject should be fed:
  - 5 oz of MiniCal
  - 2 oz of LiquiFast
  - 10 oz of SlimQuick

	MiniCal	LiquiFast	SlimQuick
Potassium (mg)	50	75	10
Protein (g)	5	10	3
Vitamin D (units)	90	100	50

- A more practical application might involve dozens of foods and nutrients rather than just three.
  - Such problems lead to systems with large numbers of variables and equations.
  - Computers or graphing calculators are essential for solving such large systems.

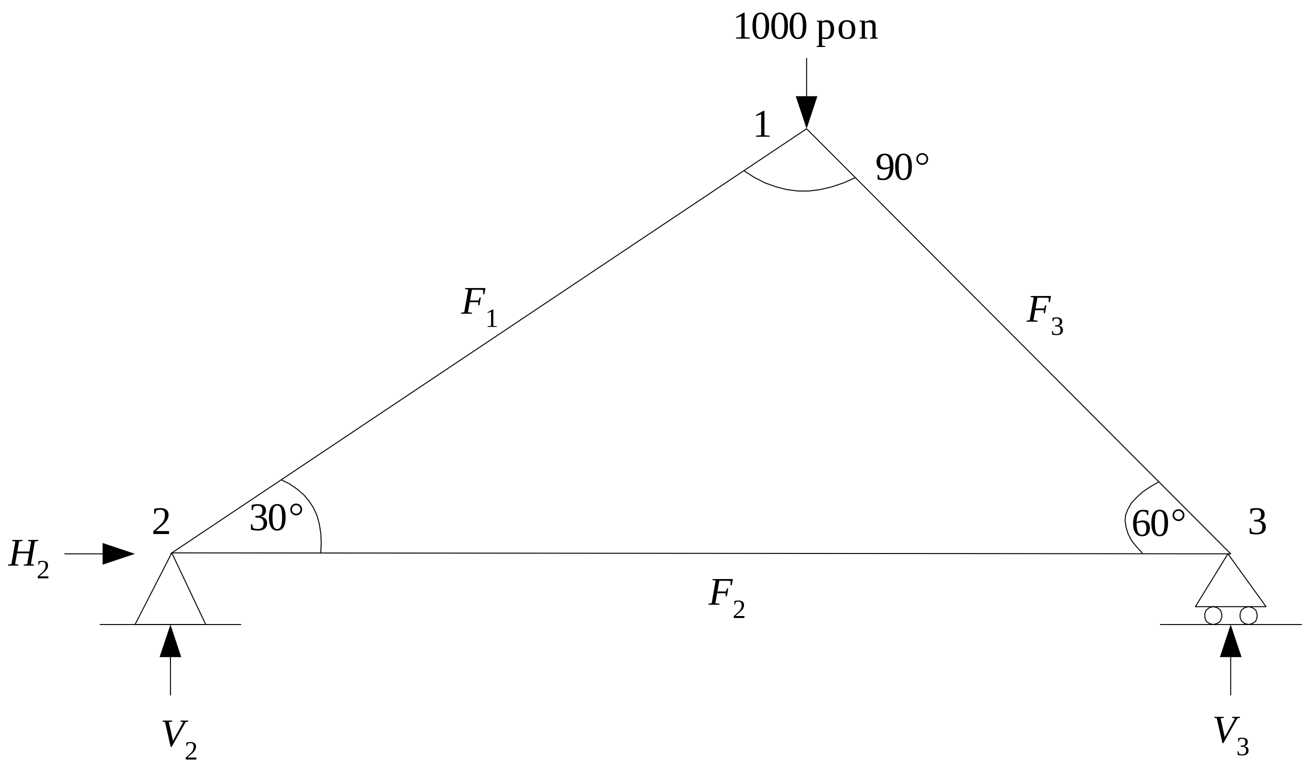
Sistem Persamaan Linier dalam bidang Teknik Sipil

Sumber: Chapra, Steven C dan Canale, Raymond P, *Numerical Methods for Engineers with Personal Computer Applications*, MacGraw-Hill Book Company

- Seorang insinyur Teknik Sipil merancang sebuah rangka statis yang berbentuk segitiga (Gambar 1). Ujung segitiga yang bersudut 30° bertumpu pada sebuah penyangga statis, sedangkan ujung segitiga yang lain bertumpu pada penyangga beroda.

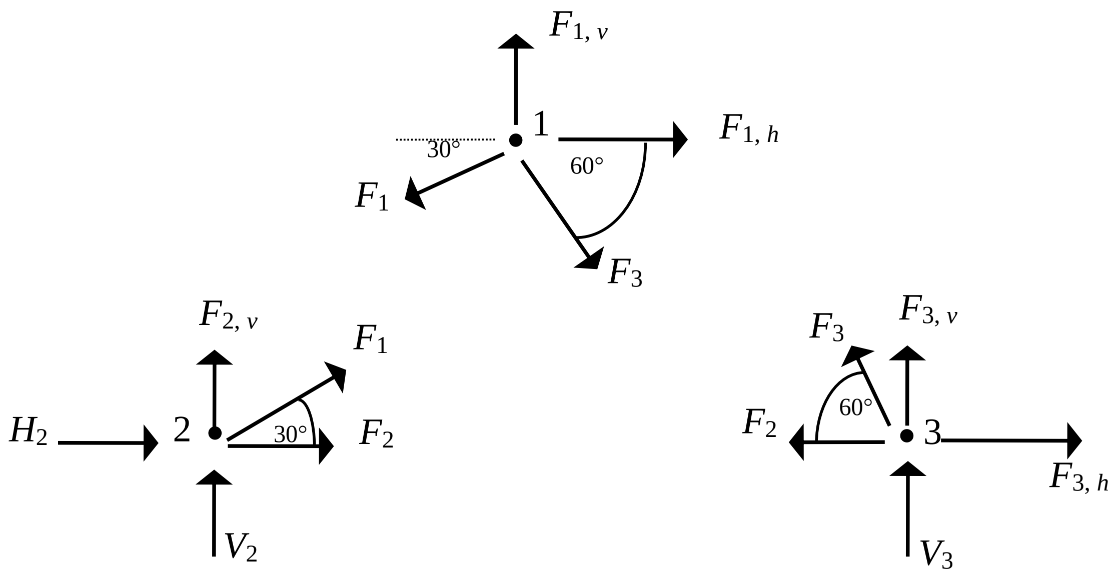
- Rangka mendapat gaya eksternal sebesar 1000 pon. Gaya ini disebar ke seluruh bagian rangka. Gaya F menyatakan tegangan atau kompresi pada anggota rangka. Reaksi eksternal (H2 , V2 , dan V3) adalah gaya yang mencirikan bagaimana rangka berinteraksi dengan permukaan pendukung.

- Engsel pada simpul 2 dapat menjangkitkan gaya mendatar dan tegak pada permukaan, sedangkan gelinding pada simpul 3 hanya menjangkitkan gaya tegak.



Gambar 1

- Struktur jenis ini dapat diuraikan sebagai sistem persamaan linier lanjar simultan. Diagram gaya-benda-bebas diperlihatkan untuk tiap simpul dalam Gambar 2.



Gambar 2

Menurut hukum Newton, resultan gaya dalam arah mendatar
Maupun tegak harus nol pada tiap simpul,
karena sistem dalam keadaan diam (statis).

Oleh karena itu, untuk simpul 1,

$$\sum F_H = 0 = -F_1 \cos 30° + F_3 \cos 60° + F_{1,h}$$
$$\sum F_V = 0 = -F_1 \sin 30° - F_3 \sin 60° + F_{1,v}$$

Untuk simpul 2,

$$\sum F_H = 0 = F_2 + F_1 \cos 30° + F_{2,h} + H_2$$
$$\sum F_V = 0 = F_1 \sin 30° - F_{2,v} + V_2$$

dan untuk simpul 3,

$$\sum F_H = 0 = -F_2 - F_3 \cos 60° + F_{3,h}$$
$$\sum F_V = 0 = F_3 \sin 60° + F_{3,v} + V_3$$

- Gaya 1000 pon ke bawah pada simpul 1 berpadanan dengan $F_{1,v}$ = -1000, sedangkan semua $F_{i,v}$ dan $F_{i,h}$ lainnya adalah nol.

- Persoalan rangka statis ini dapat dituliskan sebagai sistem yang disusun oleh enam persamaan lanjar dengan 6 peubah yang tidak diketahui:

$$\sum F_H = 0 = -F_1 \cos 30^\circ + F_3 \cos 60^\circ + F_{1,h} = -0.866F_1 + 0.5\,F_3$$
$$\sum F_V = 0 = -F_1 \sin 30^\circ - F_3 \sin 60^\circ + F_{1,v} = -0.5F_1 - 0.866\,F_3 + 1000$$
$$\sum F_H = 0 = F_2 + F_1 \cos 30^\circ + F_{2,\mathrm{h}} + H_2 = F_2 + 0.866F_1 + 0 + H_2$$
$$\sum F_V = 0 = F_1 \sin 30^\circ - F_{2,v} + V_2 = 0.5\,F_1 + V_2$$
$$\sum F_H = 0 = -F_2 - F_3 \cos 60^\circ + F_{3,h} = -F_2 - 0.5\,F_3$$
$$\sum F_V = 0 = F_3 \sin 60^\circ + F_{3,v} + V_3 = 0.866\,F_3 + V_3$$

- Keenam persamaan di atas ditulis ulang kembali dalam susunan yang teratur berdasarkan urutan peubah F_1, F_2, F_3, H_2, V_2, V_3:

$$
\begin{array}{llllllll}
-0.866F_1 & & + 0.5F_3 & & & & = 0 \\
-0.5F_1 & & - 0.866F_3 & & & & = -1000 \\
-0.866F_1 & - F_2 & & - H_2 & & & = 0 \\
-0.5F_1 & & & & - V_2 & & = 0 \\
 & - F_2 & - 0.5F_3 & & & & = 0 \\
 & & -0.866F_3 & & & - V_3 & = 0
\end{array}
$$

- atau dalam bentuk matriks:

$$\begin{bmatrix} 0.866 & 0 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.866 & 0 & 0 & 0 \\ -0.866 & -1 & 0 & -1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & -0.866 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Nilai F_1, F_2, F_3, H_2, V_2, dan V_3 yang memenuhi keenam persamaan tersebut secara simultan dapat ditemukan dengan metode eliminasi Gauss/Gauss-Jordan.